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Social Welfare and Coercion in Public Economics

by

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Extended Abstract

The social planning approach allows the policy maker to coerce individuals by using taxation and by setting a preferred level of public goods in order to maximize social welfare and to obtain the redistribution of income necessary to achieve this goal. Although those using the approach often recognize that there are problems associated with the coercion implicit in public policies, these problems are not addressed as part of the formal framework. Politics or institutions that may represent a response to coercion remain outside the analysis except for occasional ad hoc adjustments that may be made when recommendations are applied to actual situations by astute policy advisors.

There are two well-known theoretical approaches falling outside the social planning literature that deal directly with coercion inherent in public sector policies. Wicksell and Lindahl proposed that all policies be implemented using a unanimity rule. This provides a decision process resulting both in efficiency and the absence of coercion. As has been pointed out repeatedly, the process may lead to high decision costs however. Wicksell's suggestion to use modified or approximate unanimity does not come fully to grips with this problem, not being descriptive of the real world. The second approach, developed by Buchanan and Tullock, aims at finding an optimal policy rule in the face of decision costs and coercion. Like the two Scandinavian economists, they focus primarily on process and do not link their analysis to the welfare implications of particular policies.

The power of social welfare analysis derives in large part from its ability to consider the implications of specific policy choices. This allows one to compare different outcomes and to evaluate them in the context of social welfare. A purely process-oriented approach cannot accomplish this, limiting the analyst to giving advice on institutions and their design rather than on specific policies. Yet, the literature focusing on process has made an important contribution by drawing attention to the central role of coercion in public life and to the need to formally acknowledge it as a central factor in policy analysis.

This paper develops an ecumenical approach to normative policy choice which recognizes the contributions of both literatures. We examine maximization of social welfare in a framework where coercion is explicitly acknowledged as a constraint on the planner. The precise definition of coercion and its incorporation into a broader approach to social welfare are key issues in the paper. We map the trade-off between social welfare and coercion in an expanded framework, using a system with linear income taxation and public goods as an illustration. The approach allows for computation of the degree of coercion implied by standard social welfare maximization and demonstrates that the Ramsay Rule for commodity taxation must be substantially modified in an extended framework.

Key words: Coercion, optimal taxation, public goods, collective choice, unanimity, solidarity

The essential feature that defines a democratic government is voluntary agreement by members of the public to subject themselves to its coercion.

William Baumol (2003, 613)

From the point of view of general solidarity...parties and social classes should...share an expense from which they receive no great or direct benefit. Give and take is a firm foundation of lasting friendship... It is quite a different matter, however, to be forced so to contribute. Coercion is always an evil in itself and its exercise, in my opinion, can be justified only in cases of clear necessity.

Knut Wicksell (1958, 90)

1. Introduction

The social planning approach to taxation and public finance generally allows the policy maker to coerce individuals by using taxation and by setting a preferred level of public goods in order to maximize social welfare and to obtain the redistribution of income necessary to achieve this goal. Although those using the approach often recognize that there are problems associated with the coercion implicit in public policies, these problems are not addressed as part of the formal framework. Politics or institutions that may represent a response to coercion remain outside the analysis except for occasional ad hoc adjustments that may be made when recommendations are applied to actual situations by astute policy advisors.

There are two well-known theoretical approaches falling outside the social planning literature that deal directly with coercion inherent in public sector policies. Wicksell (1896) and Lindahl (1919) proposed that all policies be implemented using a unanimity rule. This provides a decision process resulting both in efficiency and the absence of coercion. As has been pointed out repeatedly(references), the process may lead to high decision costs however. Wicksell's suggestion to use modified or approximate unanimity does not come fully to grips with this problem, not being descriptive of the real world. The second approach, developed by Buchanan and Tullock (1962), aims at finding an optimal policy rule in the face of decision costs and coercion. Like the two Scandinavian economists, they focus primarily on process and do not link their analysis to the welfare implications of particular policies.

The power of social welfare analysis derives in large part from its ability to consider the implications of specific policy choices. This allows one to compare different outcomes and to evaluate them in the context of social welfare.

A purely process-oriented approach cannot accomplish this, limiting the analyst to giving advice on institutions and their design rather than on specific policies. Yet, the literature focusing on process has made an important contribution by drawing attention to the central role of coercion in public life and to the need to formally acknowledge it as a central factor in policy analysis.

This paper develops an ecumenical approach to normative policy choice which recognizes the contributions of both literatures. In this sense we are in the same tradition as Buchanan (1968), Breton (1996) and Usher (1982) who have also attempted to integrate concern with coercion into public finance and political economy, though in different ways. We examine maximization of social welfare in a framework where coercion is explicitly acknowledged as an integral part of the normative framework for policy evaluation The precise definition of coercion and its incorporation into a broader approach to social welfare optimization are key issues in the paper. We map the trade-off between social welfare and coercion in an expanded framework, using a system with linear income taxation and public goods as an illustration. The approach allows for computation of the degree of coercion implied by standard social welfare maximization and demonstrates that the Ramsay Rule for commodity taxation must be substantially modified in an extended framework.

We realize that a combination of approaches will only partially reflect the core assumptions of each. For example, our acceptance of a planner who maximizes a welfare function rather than his own utility falls outside the public choice tradition where it is consistently assumed that all actors pursue their own self-interest. Similarly, imposing criteria derived from a concern with the quality of collective choice extends the analysis beyond criteria generally accepted in much of normative public finance. We believe that our approach is justified because it allows us to address questions that cannot be readily dealt with in either of the traditional approaches.

2. The Role of Coercion in Policy Making

2.1 Where does coercion in the real world come from?

We can look at participation in a political community as an imperfect exchange, where benefits and costs are not fully matched on an individual basis (Buchanan 20XX). How does this imperfect matching arise?

- We need a public sector to provide collective goods such as security, from which everyone benefits.
- Because of jointness in supply and economies of scale, these goods and services cannot be efficiently provided in private markets.
- In addition, since unanimity doesn't work for strategic behavior reasons (Buchanan and Tullock 1962), some form of majority rule is required as long as we are interested in a democratic society.
- Majority rule of whatever form leads to coercion and thus to situations in which there will be an imperfect matching of what people pay in taxes and what they actually receive in public services.

This imperfect matching between what a citizen gets and what he or she pays is what we refer to as coercion. As Wicksell (1896) points out, this imperfect matching does not refer to, and goes beyond voluntary redistribution.

2.2 Justification for concern with coercion

People accept participation in a collectivity because they are better off with some coercion than with a state of anarchy (to put it starkly), as Baumol (2003) points out. In other words, people will accept a certain amount of coercion as a necessary cost of having a collectivity. But this is a keenly felt cost. If the cost becomes too great for too many, emigration, unrest and eventually failure of the state as a productive enterprise may occur. In more general terms, too much coercion endangers the operation of the collective choice process and the production of public services.

2.3 Our approach to modeling the role of coercion

As we noted in the Introduction, the power of social welfare analysis derives in large part from its ability to consider the implications of specific policy choices. This allows one to compare different outcomes and to evaluate them in the context of social welfare. A purely process-oriented approach - which is the way in which coercion has been incorporated into normative analysis so far - cannot accomplish this, limiting the analyst to giving advice on institutions and their design rather than on specific policies. This paper develops an ecumenical approach to normative policy choice which recognizes the contributions of both literatures.

To develop a normative approach that allows us to compare and evaluate specific policies, we proceed by imposing coercion constraints on maximization of social welfare as usually defined, investigate the nature of public polices that emerge form maximization of social welfare subject to such constraints, and compare them to policies that are consistent with the traditional social planning approach.

Is it reasonable to use coercion constraints to acknowledge the importance of coercion? Consider an analogy to modeling the social role of money. Monetary theory has tried to come to grips with the role of money in society either by putting money into the utility function (an obvious approximation to the social role of money), or by adding constraints to the specification of the economy (e.g. cash in advance constraints of Robert Clower 19XX). Our approach is analogous to the second method. We add coercion constraints to a planning problem in order to incorporate an important aspect of collective choice in a simple manner.

2.4 Why doesn't social welfare maximization take coercion into account?

Maximization of social welfare, where welfare is defined as the (possibly weighted) sum of utilities, does not deal with coercion. Although the difference between benefits and costs is reflected in individual utility, the approach posits no limits on the degree of coercion or the loss or gain in utility for particular individuals that may occur as part of a social plan. As a result, the practical importance of coercion in public life is ignored, leading to ad hoc adjustments to social plans when policy advisors are forced to take 'political realities' into account.

In addition, it should be recalled that in much applied work, social welfare analysis goes beyond strict Pareto efficiency, which is too weak to allow most social action, utilizing the Hicks-Kaldor potential compensation criterion instead. For this reason, explicit concern with coercion is justified in most practical instances.

It is important to note again that the framework we develop is *not* concerned with redistribution, a focus of many traditional social welfare optimization analyses. Distribution is an added or separate issue from the acceptability of the collective choice mechanism as Usher (1982) has emphasized. Social planning assumes that if some people pay more than they receive in services in an optimal plan, whatever that difference may be, then this is a voluntary contribution on their part for distributional reasons. (That is, the implied redistribution is acceptable as a matter of 'social solidarity' , or acceptance of the posited social welfare function). We view the implied degree of coercion as a contribution for the maintenance of a collective choice mechanism which is necessarily coercive.

3. Defining Coercion

In our approach, the social planner maximizes an unweighted sum of utilities subject to the coercion constraints and the structure of the private economy. A first step in developing such an approach is to define coercion.

There are two important choices to be made in doing so. First, coercion can be defined on an individual or a group basis. Second, coercion can be defined using utility or, as proves useful in some cases, approximating changes in utility using levels of the public good that will appear in the formal model, in a manner described below.

Concerning the first choice, applying constraints to each individual is consistent with the tradition initiated by Wicksell and Lindahl. But we also want to explore a stronger criterion that allows for a great degree of coercion. Although there isn't a complete parallel, the approach is similar to the use of the Hicks-Kaldor potential compensation criterion which allows for stronger policy judgments. We consider both cases in what follows, although we rely on a particular approach in selected instances.

The possibilities are outlined in the following table and discussed further in what follows. It is important to note that each possible definition depends on the choice of a counterfactual, and that the value or level of coercion is determined simultaneously with the choice of the fiscal system that maximizes social welfare subject to the coercion constraints.[1]

[Table 1 here]

In order to characterize coercion, we need to ask how it is perceived by the individual. Individuals are unable to adjust to given tax-prices as they would to market prices in the private economy. We relate coercion to the preceived loss in utility from this inability. This implies that we need to define a counterfactual specifying what would be possible if they could quantity adjust. The counterfactual we employ is the hypothetical level of utility or of the public good that the individual would like to have if they could quantity adjust at the tax

[1] Breton (1996) is an earlier attempt to define coercion in a related manner. Successfully dealing with coercion in actual fiscal systems is what he refers to as establishing a 'Wicksellian connection' (between what you pay and what you get).

Table 1

Alternative Definitions of Coercion

		Type of coercion constraint	
Coercion defined with respect to:	Counterfactual	*Individual (j=1,..N)*	*Aggregate*
Utility	*Utility when the level of the public good is what the individual desires at the tax price determined by the social plan*	***Case 1:*** $(V^*_j - V^e_j) \leq K_j$	***Case 2:*** $\sum_j (V^*_j - V^e_j) \leq K$
Level of public good	*Desired level of the public good compared to the supply forthcoming in the social plan*	***Case 3:*** $\lvert P_G(G^*_j - G^e)\rvert \leq K_j$	***Case 4:*** $n_1 P_G \sum_j (G^e - G^*_j) + n_2 P_G \sum_j (G^*_j - G^e) \leq K$

Notes to Table 1:

*G_j * = the level of a public good that the individual would like to the community to provide at his or her given tax price*
G^e = the level of the public service provided, assumed to be the same for all individuals
K_j = the 'degree' of coercion for citizen/taxpayer j. (In Greek, the word for coercion is 'katanagasmos'.)
K (unsubscripted) = an aggregate level of coercion
n_i = the number of taxpayer/citizens of a given type type i
P_G = marginal cost of the public good (assumed constant)
V = maximum desired utility at the individual's given tax price if that person could determine the level of the public good.*

price chosen by the planner. [2] Coercion and individualized tax-prices are thus simultaneously defined.

It should be noted that the definition we use is symmetrical. We treat people who would like less of the public good in the same way as those who get more. The first type of citizen is losing utilty because they would like a lower quantity of the good at the tax-price they face. The second type also fails to get their desired amount, but gain from the fact that they would be willing to pay more for what they receive than they are required to pay.

Although one may argue that the first type are those who are 'coerced' in the popular sense of the word, we adopt a broader perspective that relates to the overall functioning of the system. All differences between what people would like are perceived as damaging and have to recognized in chosing a fiscal system. If deviations in either direction become too large, the collective choice that is being represented indrectly (by the incorporation of coercion constraints) loses its legitimacy.[3]

3.1 Coercion defined by utility levels

To proceed further, we must be more specific about the definition of coercion. In the first case outlined in Table 1, coercion is defined for individuals by the difference between the hypothetical level V^* and the level of utility that the individual enjoys in the social planning solution V^e:

$$[V_j^*(G^*, W_j, \; t^e_j \, P_G) \; - V^e] < \; K_j, \; for \; all \; j = 1,..,N, \tag{1}$$

where

$$G^*_j = \underset{\{G\}}{arg\, max} \; V_j(G, W_j, \; t^e_j \, P_G).$$

and where

K_i = the 'degree' of coercion for citizen/taxpayer i. (We use K because in the Greek, the word for coercion is 'katanagasmos'.)
$t^e_j \, P_G$ = tax-price facing person j. The given individual tax share times the supply price of the public good G, assumed to be constant with respect to G. This is different from t^e, the chosen tax rate (or system) which determines the individual taxpayer's given tax-price.
$G_j{}^*$ = the level of a public good that the individual would like to the community to provide at his or her given tax price $t^e_j \, P_G$
G^e = the level of the public service provided, assumed to be the same for all individuals
V^* = desired utility at the given tax price if the individual could determine the level of the public good
V^e= actual utility $V^e(G^e, W_j, \; t^e_j \, P_G)$

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[2] The same type of approach is used in defining a Lindahl equilibrium. As Atkinson and Stiglitz (1980, 509) state: "The essence of the Lindahl procedure is that individuals 'demand' a total quantity of public goods on the basis of a specified distribution of the tax burden."
[3] One way of looking at this situation may be to model fairness formally. We do not follow this route here.

W_j = the given wage (or ability) of citizen/taxpayer j.

As the table illustrates, and in line with our previous discussion, we can also define coercion over all individuals together as in case 2. In the latter case, the constraint on the planner will be less severe since utility losses and gains of individuals can be offset against each other.

3.2 Coercion defined by levels of the public good

Coercion can also be defined using the level of the public good, either on an individual or on an aggregate basis. As Figure 1 shows, the difference in utility on the right side of (1) is monotonically related to the difference between the hypothetical levels of the public good and that provided by the planner, $(G^* - G^e)$. For this reason we can use this difference as an index of coercion. This will be so regardless of whether the individual prefers a level of the public good (given his or her tax price $\tau^e_j P_G$) that is higher or lower than that determined by the planner.

[Figure 1 here]

In the figure the individual is paying more for G^e than they would like at their given tax-price, and hence wants less G than is provided. But it also possible that an individual may desire more than the planner provides. To allow for both cases we may use the absolute value of the difference between hypothetical levels of G and planned levels, as shown in case 3 of Table 1. Case 4 is the analogue to case 2 where an aggregate definition of coercion is used. In working out examples, it proves convenient to use the public good formulation of coercion rather than to use utility directly.

Finally, we note that although inequality constraints are used in the definitions in Table 1, in our formal analysis we assume that equalities apply and that solutions are interior with respect to all constraints.

4. Coercion and Optimal Income Taxation with a Linear Income Tax and One Pure Public Good

The formal analysis proceeds by adding coercion constraints into a standard social planning situation. For convenience we follow Sandmo's (1998) notation.

4.1 The individual citizen – taxpayer when the government chooses fiscal policy

We assume an economy with N number of consumers-taxpayers. Each voter-taxpayer *j*, is assumed to maximize a utility function defined over a private consumption good X_j,, leisure L_i and a publicly provided good G supplied by the government:

$$U_j = U_j(X_j, L_j, G), j = 1, \ldots, N. \tag{1}$$

Utility is maximized subject to the budget constraint that consumption equals labour income after taxes have been paid. A linear tax system is assumed, that is, one with a constant marginal tax rate t and a lump sum component *a*. Let W_j denote the wage rate of consumer *j*, which is assumed to reflect *j*'s productive ability, and H_j denote his supply of labour, where $L_j + H_j = 1$, represents the normalized time endowment of *j*. The budget constraint of *j* is the written

$$X_j = (1–t)W_j(1–L_j) + a. \tag{2}$$

With $a = 0$ the tax is proportional to income, while with $a >$ (<) 0 the tax is progressive (regressive). Maximization of (1) subject to (2) yields the well-known condition that the marginal rate of substitution between leisure and consumption equals the after-tax wage rate

$$U_{jL}/U_{jX} = (1–t)W_j. \tag{3}$$

Upon solving the maximization problem we derive the consumption demand and labour supply functions of the consumer, which in general are conditional on the size of public provision. We may then write

$$X_j = X_j[(1–t)W_j,\ a,\ G] \quad and \quad H_j = H_j[(1–t)W_j,\ a,\ G] \tag{4}$$

The latter can be plagued back to the utility function to yield the indirect utility function, which depends on the net wage rate and the fiscal parameters:

$$V_j = V_j[(1–t)W_j,\ a,\ G] \tag{5}$$

Denoting the marginal utility of income by λ_j, the partial derivatives of the latter with respect to the fiscal variables are:

$$V_{jt} = –\lambda_j W_j H_j, \quad V_{ja} = –\lambda_j \quad and \quad V_{jG} = U_{jG} \qquad j=1,…,N \tag{6}$$

Moreover, we may write the marginal rate of substitution between the public good and private consumption as the ratio, which can also be thought as the marginal willingness to pay for the public good, as

$$m_j = U_{jG}/U_{jX} = V_{jG}/\lambda_j \tag{7}$$

4.2 The individual taxpayer when he or she chooses freely the level of the public

When the consumer-taxpayer is not coerced but is free to choose the public good G_j given the tax share τ_j, his optimization problem takes the form of maximizing

$$U_j = U_j(X_j\ ,\ L_j,\ G^*_j\ ),\ j = 1,\ …,\ N \tag{1'}$$

subject to the budget constraint that total expenditure for both the private and the public good equals consumer income. The tax share of j is defined as the ratio of the tax paid by j given the tax system to the total tax revenue, $\tau_j = T_j \div \Sigma_j T_j$. With $T_j = tY_j – a$, the tax share is

$$\tau_j = \frac{t^e Y_j - a}{t^e \Sigma_j Y_j - Na^e} \tag{8}$$

Let P denote the unit price of the public good. The budget constraint of consumer j is then

$$X_j + \tau_j PG_j = W_j(1 - L_j) \tag{2'}$$

Maximization of (1') subject to (2') and denoting the Lagrange multiplier by λ^*_j yields the first order conditions

$$U_{jX} = \lambda^*_j \qquad U_{jL} = \lambda^*_j W_j \quad \textit{and} \quad U_{jG} = \lambda^*_j \tau_j P.$$

Upon solving the above we derive the demand functions for X_j and G^*_j and supply of H^*_j function. Substituting those three into the utility function (1') we obtain the indirect utility function

$$V^*_j = V^*_j[W_j, \tau_j P].$$

Differentiating the above with respect to τ_j, we derive that the effect of a change in the tax share on utility is given by the expression

$$V^*_{j\tau} = -\lambda^*_j PG^*_j \tag{3'}$$

Moreover, differentiating the tax share of j τ_j with respect to the income tax rate t and the lump-sum transfer a from equation (8) and recognizing that a change in t and a affects the level of income, we obtain

$$\frac{\partial \tau_j}{\partial t} = \frac{t^2(\overline{Y}Y_{Jt} - Y_j\overline{Y_t}) + a(\overline{Y} - Y_j) + at(\overline{Y_t} - Y_{jt})}{N(t\overline{Y} - a)^2} \tag{8.1}$$

$$\frac{\partial \tau_j}{\partial a} = \frac{t(Y_{ja}\overline{Y} - Y_j\overline{Y_{ja}}) + Y_j - \overline{Y} + a(Y_{ja} - \overline{Y_a})}{N(t\overline{Y} - a)^2} \tag{8.2}$$

where $\overline{Y} = \Sigma_j Y_j \div N$ denotes the mean income. Upon combining (3') with (8.1) and (8.2) we can identify the effect of a change in the tax rate and the lump-sum transfer on utility.

4.3 *Social welfare maximization under aggregate coercion*

In choosing fiscal policy instruments, the coercion-constrained planner is assumed to maximize the sum of individual utility functions, which we refer to generally as coercion-constrained welfare:

$$S = \Sigma_j V_j \tag{9}$$

The budget constraint of the government requires that total tax revenue equals the expenditure for the public good

$$t\Sigma_j W_j H_j - Na = PG \tag{10}$$

In addition to the usual budget constraint the planner faces coercion constraints. To begin, it is useful to look at case 2 in Table 1 (aggregate coercion defined using utility). Let κ denote the Lagrangean multiplier of the coercion constraint. In general, the optimization problem now becomes to maximize

$$\mathcal{L} = \Sigma_j V_j + \mu\,[t\Sigma_j W_j H_j - Na - PG] + \kappa[\Sigma_j (V^*_j - V_j) - K] \tag{11}$$

Differentiating the above with respect to t, a and G and substituting from (6) and (3') we obtain respectively

$$(1-\kappa)\Sigma_j\lambda_j W_j H_j + \kappa\Sigma_j\lambda^*_j\, P\, G^*_j\ (\partial\tau_j/\partial t) = \mu\,[\Sigma_j W_j H_j + t\Sigma_j W_j(\partial H_j/\partial t)] \tag{11.1}$$
$$(1-\kappa)\Sigma_j\lambda_j - \kappa\Sigma_j\lambda^*_j\, PG^*_j\ (\partial\tau_j/\partial a) = \mu\,[N - t\Sigma_j W_j(\partial H_j/\partial a)] \tag{11.2}$$
$$(1-\kappa)\Sigma_j\lambda_j m_j = \mu\,[P - t\Sigma_j W_j(\partial H_j/\partial G)] \tag{11.3}$$

Equations (11.1) and (11.2) feature two new important elements that are absent from the traditional optimal taxation - social planning analysis: (a) the translation of tax structure into the tax price - here shown as $(\partial\tau_j/\partial t)$ and $(\partial\tau_j/\partial a)$; (b) the translation of the tax price into the demand for G - here shown as $\lambda^*_j\, P\, G^*_j$ - see equation (3') above. These two elements always appear in one form or another in all coercion constrained planning problems.

There are many ways to perform the translation of t into τ. We assume that the implied average tax price is also the one that applies to marginal changes in public services when viewed from the perspective of each individual citizen-taxpayer.

Using the covariance formula $\sigma^2_{\lambda m} = (\Sigma_j\lambda_j m_j/N) - (\Sigma_j\lambda_j/N)(\Sigma_j m_j/N)$ and setting $\lambda = \Sigma_j\lambda_j/N$ the mean value of the marginal utility of income (11.3) is rewritten as

$$(1-\kappa)[N\sigma^2_{\lambda m} + \lambda\Sigma_j m_j] = \mu\,[P - t\Sigma_j W_j(\partial H_j/\partial G)].$$

Denoting $m = \Sigma_j m_j/N$, the mean value of the marginal rate of substitution between the public good and private consumption, and manipulating the above becomes

$$(1-\kappa)(\Sigma_j m_j)(\sigma^2_{\lambda m}/\lambda m)] = (\mu/\lambda)\,[P - t\Sigma_j W_j(\partial H_j/\partial G)]$$

Let $\delta \equiv \sigma^2_{\lambda m}/\lambda m$ the normalized covariance between the marginal rate of substitution and the marginal utility of income reflects the distributional characteristics of the public good, we derive

$$\left(\Sigma_j m_j\right)(1+\delta)(1-\kappa) = \frac{\mu}{\lambda}\left(P - t\Sigma_j W_j \frac{\partial H_j}{\partial G}\right). \tag{12}$$

Equation (12) represents the condition for optimal provision of the public good when the government cannot breach a coercion constraint. The left-hand-side represents the social marginal benefit from public provision in the presence of the coercion constraint. It is the product of three terms: (a) the sum of the marginal rates of substitution between the private and the public good; (b) term $1+\delta$, which adjusts the sum of MRSs for the distributional characteristics of its provision – captured by the expression $\sigma^2_{\lambda m}/\lambda m$; (c) the term $(1-\kappa)$, which reflects the effect of the coercion constraint. The more stringent the constraint, that is, the less tolerant the planner is of coercion, the higher the magnitude of κ and hence the lower the size of the term $(1-\kappa)$. Thus, in the presence of aggregate coercion, optimality in the provision of the public good requires the adjustment of the sum of the marginal rates of substitution for the combined effect of distribution and coercion.

The right-hand-side represents the social marginal cost of public provision. It is the product of two components, familiar from the standard social welfare maximization case. (a) The marginal valuation of government revenue, μ/λ; see Sandmo (1998). (b) The net marginal rate of transformation of the public good, $P - t\sum_j W_j(\partial H_j/\partial G)$, which equals the unit cost of production of the public good adjusted for the effects of public provision on income and therefore income taxation (and can therefore increase or decrease the revenue required for an extra unit of the public good).

For optimality, the marginal benefit of the left-hand-side must be equal to the marginal cost of the right-hand-side.

The standard rule of optimal public good provision can be derived as a special case of equation (12) when it is assumed that the planner does not observe a coercion constraint, so that $\kappa=0$. In this case (see Sandmo 1998),

$$\left(\Sigma_j m_j\right)(1+\delta) = \frac{\mu}{\lambda}\left(P - t\Sigma_j W_j \frac{\partial H_j}{\partial G}\right). \tag{12'}$$

Comparing (12) to (12') we observe that since κ is positive, $\sum_j m_j$ must be large to maintain the equality in (12), and thus G and t must be lower than in a standard planning solution. Intuitively, and assuming δ is positive, the declining marginal rate of substitution between public and private goods for all individuals implies that on average, those who are less coerced when G is increased gain less than do those who want less G when G is decreased.

We can define the coercion adjusted marginal cost of funds (MCF) that is appropriate for policy analysis when one is formally cognizant of coercion. This is the term: $\mu/[\lambda(1-\kappa)]$. It must be higher than when coercion is ignored. But note that the MCF as a concept is still relevant here.

As far as deriving the optimal income tax rate is concerned, we can proceed as follows. Multiplying (11.1) by (1/N) and (11.2) by $(\sum_j W_j H_j/N^2)$ and subtracting the latter from the former we have

$$(1-\kappa)\left(\frac{\Sigma_j \lambda_j W_j H_j}{N} - \frac{\Sigma_j \lambda_j}{N}\frac{\Sigma_j W_j H_j}{N}\right) =$$
$$t\frac{\mu}{N}\left[\left(\Sigma_j W_j \frac{\partial H_j}{\partial t}\right) + \left(\Sigma_j W_j \frac{\partial H_j}{\partial a}\right)\left(\frac{\Sigma_j W_j H_j}{N}\right)\right] - \frac{\kappa}{N}\Sigma_j \lambda *_j \, PG*_j \left(\frac{\partial \tau_j}{\partial t} + \frac{\partial \tau_j}{\partial a}\overline{Y}\right).$$

Recalling the Slutsky decomposition, $\partial H_j/\partial t = s_j - W_j H_j(\partial H_j/\partial a)$ and using again the covariance formula (where $Y_j = W_j H_j$) , the previous equation yields

$$(1-\kappa)\sigma^2{}_{\lambda Y} =$$
$$t\frac{\mu}{N}\left[\Sigma_j W_j s_j - \Sigma_j W_j H_j \frac{\partial H_j}{\partial a} + \left(\Sigma_j W_j \frac{\partial H_j}{\partial a}\right)\left(\frac{\Sigma_j W_j H_j}{N}\right)\right] - \frac{\kappa}{N}\Sigma_j \lambda *_j \, PG*_j \left(\frac{\partial \tau_j}{\partial t} + \frac{\partial \tau_j}{\partial a}\overline{Y}\right). \quad (13)$$

The term $\overline{WS} = \frac{\Sigma_j W_j s_j}{N}$ is the mean substitution effect of taxation on labour supply, which is negative. The covariance term $\sigma^2{}_{Ya} = \frac{1}{N}\left[\Sigma_j W_j H_j \frac{\partial H_j}{\partial a} - \left(\Sigma_j W_j \frac{\partial H_j}{\partial a}\right)\left(\frac{\Sigma_j W_j H_j}{N}\right)\right]$ shows the relationship between income and the income effect of taxation; it is non-negative when those with large income are characterised by small income effects on labour supply.

From equation (3') we have that $V*_{j\tau} = -\lambda *_j PG*_j \equiv \psi_j$ is the marginal utility of the tax share. The quantity $q_j \equiv \frac{\partial \tau_j}{\partial t} + \frac{\partial \tau_j}{\partial a}\overline{Y}$ captures the marginal tax share, that is, the total change in the tax share of j when the tax rate and the lump-sum transfer change. We may then write

$$\Sigma_j \lambda *_j \, PG*_j \left(\frac{\partial \tau_j}{\partial t} + \frac{\partial \tau_j}{\partial a}\overline{Y}\right) = \Sigma_j \psi_j q_j \,.$$

Using the covariance formula we have $\Sigma_j \psi_j q_j = N\sigma^2{}_{\psi q} + N\overline{\psi}\overline{q}$, where $\overline{\psi}$ and $\overline{q}$ denote the mean values of ψ_j and q_j respectively and the covariance term $\sigma^2{}_{\psi q}$ shows the relationship between the marginal utility of the tax share and the marginal tax share. Its value depends on the size of the parameters of the utility function and is therefore an empirical matter. However, if one presumes that tax payers who experience a large increase in their tax shares will also experience a significant fall in utility, $\sigma^2{}_{\psi q}$ will be negative. From (8.1) and (8.2) we obtain that

$$\Sigma_j \frac{\partial \tau_j}{\partial t} = \Sigma_j \frac{\partial \tau_j}{\partial a}\overline{Y} = 0,$$

and thus $\overline{q} = 0$. Substituting back into (13) and collecting terms we derive the optimal income tax rate under a coercion constraint:

$$t = \frac{(1-\kappa)\sigma^2{}_{\lambda Y} - \kappa\sigma^2{}_{\psi q}}{\mu\left(\overline{WS} - \sigma^2{}_{Ya}\right)}. \quad (14)$$

The $\sigma^2_{Y\lambda}$ covariance term in the numerator shows the relationship between the marginal utility of income and income from work and reflects the distributional effects of income taxation. Since the higher the level of income the lower its marginal utility, $\sigma^2_{Y\lambda}$ is negative. The bracketed term in the denominator reflects the efficiency effects of distortionary income taxation.

The optimal tax rate is decreasing in the marginal utility of the coercion constraint κ. That is, other things being equal, the more stringent the coercion constraint – that is, the less tolerant of the coercion are individuals –the lower the optimal income tax rate. Given that both $\sigma^2_{Y\lambda}$ and the denominator are negative, the optimal tax rate will be higher if the $\sigma^2_{\psi q}$ covariance is negative, that is, if the marginal utility of the tax share falls when the tax share rises, a result which accords well with intuition.

The standard linear optimal tax rate can be obtained as a special case of (14) when the coercion constraint is absent - see for example, Sandmo (1983). Formally, the Lagrangean multiplier of the coercion constraint is set at zero, $\kappa=0$, so that

$$t = \frac{\sigma^2_{\lambda Y}}{\mu\left(\overline{WS} - \sigma^2_{Ya}\right)} \quad . \tag{14'}$$

Thus by comparing (14) and (14') we see that the more general formulation of the optimal income tax rate features two new terms in comparison to the standard formula: (i) the marginal utility of the coercion constraint κ, and (ii) the covariance of marginal utility of the tax share and the marginal tax share, $\sigma^2_{\psi q}$.

For completeness it is worthwhile to briefly contrast this with the opposite extreme, when citizens do not tolerate *any* coercion. Formally, the size of coercion K is now set at zero:

$$\Sigma_j (V^*_j - V_j) = 0 \,.$$

In this case each citizen/taxpayer j consumes the size of the public good which maximizes his or her utility given their tax share. However, attaining equilibrium requires that all individuals consume voluntarily the same level of the public good; otherwise their consumption plans are incompatible. This is accomplished in a Lindahl equilibrium. A Lindahl equilibrium is defined as the set of tax shares where (a) given their tax shares all taxpayers demand the same level of the public good, $G^*_j = G_L$ for all j; and (b) the individual tax shares add-up to unity, $\Sigma_j \tau_j = 1$.

Algebraically, G^*_j is found from maximizing individual utility (1') subject to the budget constraint (2') which is defined over the tax price $\tau_j P$; this yields the demand $G^*_j= G^*_j(\tau_j P, W_j)$. Setting $G^*_j(\tau_j P, W_j)= G_L$ and inverting the latter we have $\tau_j = \tau_j (G_L, P, W_j)$. Upon substituting into $\Sigma_j \tau_j = 1$ and solving for G_L. $G_l = G_l(P, \Sigma_j W_j)$. The latter can then be substituted back to V^*_j to derive the level of individual utility under free choice of the public good.

5.2 Individual coercion constraints

When the planner is constrained on how much it can coerce each individual taxpayer separately, the Lagrangean of the social welfare maximization problem is written as

$$\mathcal{L} = \sum_j V_j + \mu\,[t\sum_j W_j H_j - Na - P_G G] + \sum_j \kappa_j\,(V^*_j - V^e_j - K_j). \tag{15}$$

Differentiating the above with respect to *t*, *a* and *G* we obtain respectively

$$\sum_j (1-\kappa_j)\lambda_j W_j H_j + \sum_j \kappa_j \lambda^*_j\, P_G\, G^*_j\,(\partial\tau_j/\partial t) \;=\; \mu\,[\sum_j W_j H_j + t\sum_j W_j(\partial H_j/\partial t)] \tag{15.1}$$

$$\sum_j (1-\kappa_j)\lambda_j - \sum_j \kappa_j \lambda^*_j\, P_G\, G^*_j\;(\partial\tau_j/\partial a) \;=\; \mu\,[N - t\sum_j W_j(\partial H_j/\partial a)] \tag{15.2}$$

$$\sum_j (1-\kappa_j)\lambda_j m_j = \mu\,[P_G - t\sum_j W_j(\partial H_j/\partial G)] \tag{15.3}$$

These equations are considerably more complicated than what we have encountered before because they feature three frequency distributions, κ_j, λ_j, and m_j. We have then to work with the covariance of all three variables.

The left-hand-side of (15.3) is written as

$$\sum_j (1-\kappa_j)\lambda_j m_j = \sum_j \kappa_j \lambda_j - \sum_j \kappa_j \lambda_j m_j\,.$$

Recall (where κ, λ, and m are the means of κ_j, λ_j, and m_j respectively),

$$Cov(\kappa_j \lambda_j) = (1/N)\sum_j (\lambda_j - \lambda)(m_j - m) \quad \text{and} \quad \text{cov}(\kappa_j \lambda_j m_j) = (1/N)\sum_j (\kappa_j - \kappa)(\lambda_j - \lambda)(m_j - m).$$

Manipulating, the above yields

$$\sum_j \lambda_j m_j = N\sigma^2_{\lambda m} + \lambda \sum_j m_j \quad \text{and} \quad \sum_j \kappa_j \lambda_j m_j = [\kappa\lambda \sum_j m_j + N(\kappa\sigma^2_{\lambda m} + \lambda\sigma^2_{\kappa m} + \sigma^2_{\kappa\lambda j} + \sigma^2_{\kappa\lambda m})].$$

Substituting into (15.3) and manipulating we obtain

$$\left(\sum_j m_j\right)\lambda\left[1-\kappa+\frac{\sigma^2_{\lambda m}}{\lambda m}-\kappa\left(\frac{\sigma^2_{\lambda m}}{\lambda m}+\frac{\sigma^2_{\kappa\lambda}}{\kappa\lambda}+\frac{\sigma^2_{\kappa m}}{\kappa m}+\frac{\sigma^2_{\kappa\lambda m}}{\kappa\lambda m}\right)\right]=\mu\left(P_G - t\sum_j W_j\frac{\partial H_j}{\partial G^e}\right).$$

Recall that $s_{\lambda m} \equiv \sigma^2_{\lambda m}/\lambda m\ (\equiv\delta)$, denotes the normalized covariance between the marginal rate of substitution and the marginal utility of income. Similarly, we may define $s_{\kappa\lambda} \equiv \sigma^2_{\kappa\lambda}/\lambda\kappa$ as the normalized covariance between coercion and the marginal utility of income; $s_{\kappa m} \equiv \sigma^2_{\kappa m}/\kappa m$ as the normalized covariance between coercion and the marginal rate of substitution; and $s_{\kappa\lambda m} \equiv \sigma^2_{\kappa\lambda m}/\kappa\lambda m$ as the normalized covariance between coercion, the marginal utility of income and the marginal rate of substitution. Putting these definitions into the first order condition we have

$$\left(\sum_j m_j\right)[1+\delta-\kappa-\kappa(1+\delta+s_{\lambda\kappa}+s_{\kappa m}+s_{\kappa\lambda m})]=\frac{\mu}{\lambda}\left(P_G - t\sum_j W_j\frac{\partial H_j}{\partial G^e}\right).$$

Setting the sum of the normalized covariance terms as $\phi \equiv s_{\lambda m} + s_{\kappa\lambda} + s_{\kappa m} + s_{\kappa\lambda m}$, the first order condition can be written more compactly as

$$\left(\sum_j m_j\right)\left[(1+\delta)(1-\kappa)-\kappa\phi\right]=\frac{\mu}{\lambda}\left(P_G - t\sum_j W_j\frac{\partial H_j}{\partial G^e}\right). \tag{16}$$

The left-hand-side of (16), which shows the marginal benefit from the public good, is the product of the sum of marginal rates of substitution times the adjustment for the combined effect of the distributional characteristics of the public good and the effects of coercion. In the present case of the individual coercion constraints, the adjustment for coercion contains two elements, (i) the average effect of coercion captured by the term $(1+\delta)(1-\kappa)$, a term that also features in the previous case of aggregate coercion, and (ii) the distributional characteristics of coercion captured by the term $\kappa\phi$, which corrects the aggregate term for the distributional characteristics of coercion. The right-hand-side of equation (16) is already familiar: it is the product of the marginal valuation of government revenue times the net marginal rate of transformation of the public good.

The solution in (16) differs from the solution in (12) - where coercion is defined on an aggregate basis - by the new term $-\kappa\phi$, reflecting the fact that not just aggregate coercion matters but also its distribution. Now the benefit from public provision falls:

> (i) If the rich (low λ) have more social solidarity (low κ), then $\sigma^2_{\kappa\lambda} > 0$ and $s_{\kappa\lambda} > 0$, because the rich tend to benefit more from public provision.
>
> (ii) If those who value public goods less (low m) have more social solidarity (low κ), then $\sigma^2_{\kappa m} > 0$ and $s_{\kappa m} > 0$, because of the coercion from public provision
>
> (iii) If the rich (low λ) also value public goods less (low m) and have more social solidarity (low κ), then $\sigma^2_{\kappa\lambda m} > 0$ and $s_{\kappa\lambda m} > 0$, because the previous two effects are compounded

In these cases, G and t in the solution (16) will be less than when coercion is defined in an aggregate sense. That is, the Hicks-Kaldor type of solution for a coercion-constrained optimum involves more coercion and more spending than when coercion is defined on an individual basis. Of course, it could go the other way in principle, and it will be interesting to try and figure out in practice which case is likely to apply.

In order to derive the optimal income tax rate we work in a manner similar to the one before. Multiplying (15.1) by (1/N) and (15.2) by $(\Sigma_j W_j H_j/N^2)$ and subtracting the latter from the former we have

$$\left(\frac{\Sigma_j\lambda_j W_j H_j}{N}-\frac{\Sigma_j\lambda_j}{N}\frac{\Sigma_j W_j H_j}{N}\right)-\left(\frac{\Sigma_j\kappa_j\lambda_j W_j H_j}{N}-\frac{\Sigma_j\kappa_j\lambda_j}{N}\frac{\Sigma_j W_j H_j}{N}\right)=$$
$$t\frac{\mu}{N}\left[\left(\Sigma_j W_j\frac{\partial H_j}{\partial t}\right)+\left(\Sigma_j W_j\frac{\partial H_j}{\partial a}\right)\left(\frac{\Sigma_j W_j H_j}{N}\right)\right]+\frac{1}{N}\Sigma_j\kappa_j(-\lambda^*_j\,PG^*_j)\left(\frac{\partial\tau_j}{\partial t}+\frac{\partial\tau_j}{\partial a}\overline{Y}\right). \qquad (17)$$

The left-hand-side involves the frequency distributions of three variables, the individual coercion constraint κ_j, the marginal utility of income λ_j, and income Y_j. Similarly, the right-hand-side features the individual coercion constraint κ_j, the marginal utility of the tax share $\psi_j = -\lambda^*_j PG^*_j$, and the marginal tax share, $q_j=[(\partial\tau_j/\partial t)+(\partial\tau_j/\partial a)\ \overline{Y}]$, as well as the effect of income tax on labour supply. We may then apply again the formula for the covariance of three variables.

The following notation is used: $\sigma^2_{\kappa\lambda Y}$ = covariance of κ_j, λ_j and Y_j; $\sigma^2_{\kappa\psi q}$ = covariance of κ_j, ψ_j and q_j; $\sigma^2_{\kappa Y}$ = covariance of κ_j and Y_j; $\sigma^2_{\psi q}$ = covariance of ψ_j and κ_j; $\sigma^2_{\kappa q}$ = covariance of κ_j; and q_j; and κ, λ, ψ and q are the mean values of κ_j, λ_j, ψ_j and q_j respectively. Recalling the Slutsky disaggregation and manipulating, we obtain the formula for the coercion-constrained optimal income tax rate as follows:

$$t = \frac{(1-\kappa)\sigma^2_{\lambda Y} - \kappa\sigma^2_{\psi q} - \lambda\sigma^2_{\kappa Y} - \psi\sigma^2_{\kappa q} - \sigma^2_{\kappa\lambda Y} - \sigma^2_{\kappa\psi q}}{\mu\left(\overline{WS} - \sigma^2_{Ya}\right)}. \tag{18}$$

The optimal income tax rate depends as usual on the income distribution effect of taxation (captured by $\sigma^2_{\lambda Y}$) and the efficiency effect of taxation on labour (shown by the denominator). In common with the case of aggregate coercion, it also depends on the relationship between the marginal utility of the tax share and the marginal tax share $\sigma^2_{\psi q}$. If a high marginal tax share (high q) is associated with a large fall in utility (falling ψ), $\sigma^2_{\psi q}$. < 0, and the optimal tax rate will be lower than otherwise.

In addition, the optimal tax rate depends on the distribution effects of coercion as the remaining four covariance terms make clear. Specifically, the optimal tax rate will be lower:

(i) if the rich (high Y) value social solidarity more (low κ) then $\sigma^2_{\kappa Y} < 0$; the reason is that they will benefit mostly from a high tax rate

(ii) if taxpayers who experience a high change in the tax share (high q) have low social solidarity (high κ), then $\sigma^2_{\kappa q} > 0$ (since ψ is negative); the reason is that a higher tax rate increases coercion.

(iii) if the rich (high Y and low λ) have more social solidarity (low κ), then $\sigma^2_{\kappa\lambda Y} < 0$

(iv) if $\sigma^2_{\kappa\psi q} < 0$, which is the case when (a) those who experience a small change in the tax share suffer a large utility loss also have less solidarity, or (b) those who experience a large change in the tax share suffer a large utility loss also have more solidarity.

6. The Trade-off Between Social Welfare and Coercion in Income Taxation

One of the interesting problems to investigate is the trade-off between social welfare and the degree of coercion. The trade-off arises because coercion constraints limit the range of choice of the planner. To allow a closed form solution for the trade-off, we assume utility functions have a Cobb-Douglas form and we adopt an aggregate definition of coercion based on the level of the public good (case 4 in Table 1).

The utility function of taxpayer j is written as

$$U_j = \alpha log X_j + \beta log L_j + \gamma_j log G. \tag{19}$$

Consumers are assumed to have identical tastes for the private good and leisure but different tastes for the public good, that is, $\alpha_j = \alpha, \beta_j = \beta_i$ and $\gamma_j \neq \gamma_i$, for $j \neq i$. For algebraic tractability we also assume that a proportional income tax system is in operation with a single income tax rate t. The budget constraint of the j taxpayer is then written as

$$X_j = (1-t)W_j(1-L_j). \tag{20}$$

Maximizing the utility function (1') subject to the budget constraint (2') treating t and G as set exogenously by the planner, solving for X_j and L_j and substituting back we derive the following expressions for the partial derivatives of income and the indirect utility function with respect to t and G:

$$Y_j = \alpha W_j / (\alpha+\beta) \qquad Y_{jt} = 0 \qquad Y_{jG} = 0$$
$$V_{ja} = -\lambda_j = -(\alpha+\beta)/W_j \qquad V_{jt} = -\alpha/(1-t) \qquad V_{jG} = \gamma_j\,; \qquad j=1,\ldots,N$$

The problem of the social planner is then to find the levels of t and G which maximize the social welfare function $\sum_j V_j$ subject to the budget constraint $t\sum_j W_j H_j = PG$. After the relevant manipulations, and denoting the mean value of γ_j by $\bar{\gamma}$ the socially optimal (OT) levels of the tax rate and the public good are given by

$$G^O = \frac{\alpha}{\alpha+\beta}\frac{\bar{\gamma}}{\alpha+\bar{\gamma}}\frac{\Sigma_j W_j}{P} \tag{21.1}$$

and

$$t^O = \frac{\bar{\gamma}}{\alpha+\bar{\gamma}} \quad . \tag{21.2}$$

On the other hand, if the taxpayer is free to choose the level of the public good G that he would like the society to make available to him given his tax share, his utility maximization problem is to maximize

$$U_j = \alpha log X_j + \beta log L_j + \gamma_j log G_j \tag{19'}$$

subject to the budget constraint

$$W_j(1-L_j) = X_j + \tau_j P G_j\,. \tag{20'}$$

Solving the latter yields

$$X_j = \alpha W_j; \qquad N_j = 1-\beta \qquad \text{and} \qquad G^*_j = \frac{\gamma_j W_j}{\tau_j P} \quad . \tag{21'}$$

Obviously G^*_j, differs across j = 1,…,N. Recalling that the tax share of j is $\tau_j = tY_j/\Sigma_j tY_j = W_j/\Sigma_j W_j$, the latter yields

$$G^*_j = \frac{\gamma_j \Sigma_j W_j}{P}. \tag{22}$$

Note that with Cobb-Douglas utility, and unlike the usual social welfare maximization solution, the level of the public good desired at the given tax-price does not depend on the preference coefficients for private consumption and leisure, but only on the tastes for the public good.

To keep the analysis simple, let us divide the population into two groups - 1 and 2 - with taste parameters for the public good γ_1 and γ_2 , and with wage rates W_1 and W_2. The numbers of consumers in the two groups are N_1 and N_2. It follows from (21) that the corresponding demand functions for the public good are

$$G^*_1 = \frac{\gamma_1(N_1W_1 + N_2W_2)}{P} \qquad \text{and} \qquad G^*_2 = \frac{\gamma_2(N_1W_1 + N_2W_2)}{P}. \tag{22'}$$

However, only a single level of the public good G is provided by the government. Assume that taxpayers of type 1 are forced to consume a quantity of the public good larger than the quantity that they would have chosen freely, that is, $G > G^*_1$, and that taxpayers of type 2 are forced to consume a quantity of the public good smaller than the freely chosen quantity, that is, $G < G^*_2$.

Suppose now that the government is not free to coerce taxpayers at will but it faces the constraints that taxpayers will tolerate coercion only up to a certain level. Speaking informally, the government is now subject to the coercion constraint that taxpayers of type 1 are not to be coerced to consume "too much" and that taxpayers of type 2 are not to be coerced to consume "too little". Formally,

$$(G - G^*_1)P \leq K_1 \quad \text{and} \quad (G^*_2 - G)P \leq K_2 \ .$$

Note that in general it may be that $K_1 \neq K_2$. Summing the coercion constraints over the two groups, the aggregate coercion constraint when it just bites is written as

$$N_1(G - G^*_1)P + N_2(G^*_2 - G)P \leq N_1K_1 + N_2K_2 . \tag{23}$$

Substituting from (21'), setting $K = N_1K_1 + N_2K_2$, and manipulating we find the size of the public when the coercion constraint described is operative[4]:

$$G^C = \frac{(N_1W_1 + N_2W_2)(N_2\gamma_2 - N_1\gamma_1) - K}{(N_2 - N_1)P} . \tag{24.1}$$

For the latter to be positive it must be

$$N_2 - N_1 > 0 \qquad \text{and} \qquad N_2\gamma_2 - N_1\gamma_1 > K/(N_1W_1 + N_2W_2)$$

---

[4] In the present setting, the government possesses only two instruments, the income tax rate, t, and the size of public provision, G. As the two depend on each other through the budget constraint, there is a single free instrument, whose value is found by solving the aggregate coercion constraint. In the more general case of more than two instruments, their values are found by maximizing the social welfare function subject to the budget constraint and the coercion constraint.

or

$N_2 - N_1 < 0$ and $N_2\gamma_2 - N_1\gamma_1 < K/(N_1W_1+N_2W_2)$.

For ease of exposition, in what follows we assume that the former two inequalities hold. Substituting the solution in (22.1) into the budget constraint of the government

$$PG = t\left(\frac{N_1\alpha W_1}{\alpha+\beta} + \frac{N_2\alpha W_2}{\alpha+\beta}\right),$$

we find the income tax rate when the coercion constraint is binding is

$$t^C = \frac{\alpha+\beta}{\alpha}\frac{(N_2\gamma_2 - N_1\gamma_1)(N_1W_1+N_2W_2) - K}{(N_2-N_1)(N_1W_1+N_2W_2)} \quad . \tag{24.2}$$

It is immediately seen that the income tax rate varies inversely with the size of coercion. Intuitively, the less coercion the society tolerates, the smaller the income tax rate (and the smaller the size of public provision). In addition, the higher the taste parameter of those in group 2, who have more intense preferences for the public good, the higher the tax rate. On the contrary the lower the intensity of preferences for the public good by group G, the lower the tax rate.[5]

However, note the discontinuity: t^C is defined for strictly positive values of K, when the society does not tolerate any coercion the tax rate falls to zero. Thus,

For $K = 0 \Rightarrow t = 0$
For $K > 0 \Rightarrow t = t^C$ as described in (24).

From (21.2) and (23.2) we can find the value of the coercion constraint K which is implied by the standard optimal tax - optimal public provision problem, that is, when the government ignores the coercion implied by its actions. Setting $t^C = t^O$, denoting the mean value of γ_j by $\bar{\gamma}=(N_2\gamma_2+N_1\gamma_1)/(N_2+N_1)$ and solving we derive:

$$K_{OT} = (N_1W_1+N_2W_2)\left[N_2\gamma_2 - N_1\gamma_1 - \frac{\alpha}{\alpha+\beta}\frac{\bar{\gamma}}{\alpha+\bar{\gamma}}(N_2-N_1)\right]. \tag{25}$$

The latter varies proportionately with total income, $\sum_j W_j$, that is, in the standard OT problem the higher the aggregate productivity the higher the level of the coercion that the government imposes to taxpayers. However, the rest of comparative static properties have ambiguous signs.

[5] Finally, to complete the analysis we turn to the level of public provision when $K=0$, that is, when taxpayers do not tolerate the government to coerce them. As already explained, equilibrium requires that all taxpayers consume the same level of G^L under the Lindahl equilibrium. Inserting $G^*_j = G_L$ in (21') and substituting the resulting expression for τ_j into $\sum_j \tau_j = 1$ we derive $G^L = (\bar{\gamma}\bar{W} + \sigma^2_{\gamma W})(N_1+N_2)/P$.

Plugging (24.1) and (24.2) into the individual utility function (19) and summing over all individuals we derive social welfare (the sum of individual utility functions) as a function of aggregate coercion - what we shall call *'coercion-constrained social welfare'*

$$S = \Sigma_j V_j = \Sigma_j V_j(K) \Rightarrow$$

$$S(K) = \alpha\{(N_1+N_2)[\log\alpha(1-t^C(K)) - \log(\alpha+\beta)] + N_1\log W_1 + N_2\log W_2\}$$
$$+(N_1+N_2)[\log\beta - \log(\alpha+\beta)] + (N_1\gamma_1 + N_2\gamma_2)\log G^C(K). \qquad (26)$$

Differentiating with respect to K, using (23.1) and (23.2) and rearranging we have

$$\frac{dS}{dK} = \frac{(\alpha+\beta)(\alpha+\bar{\gamma})}{\alpha}\frac{K_{OT}-K}{(N_2-N_1)^2(1-t^C)\overline{\overline{W}}PG^C} \qquad (27)$$

and

$$\frac{d^2S}{dK^2} = \frac{1}{(N_2-N_1)\overline{\overline{W}}}\frac{dt^C}{dK}\frac{1}{(1-t^C)^2} + \frac{N_1\gamma_1+N_2\gamma_2}{(N_2-N_1)P}\frac{dG^C}{dK}\frac{1}{(G^C)^2} < 0\,. \qquad (28)$$

The latter inequality follows from differentiating (24.1) and (24.2).

We then see that the trade-off between coercion-constrained welfare and the size of coercion is a concave function of K as illustrated in Figure 2. Coercion-constrained welfare is shown on the vertical axis and the degree of assumed aggregate coercion is shown on the horizontal. (The relationship between the size of government and coercion will have a similar general shape).

[Figure 2 here]

Coercion-constrained social welfare is increasing in K for $K < K_{OT}$. Analytically, starting from low levels of coercion, the higher the coercion taxpayers tolerate the easier it is for the government to implement the traditional social welfare maximization solution.

The upward sloping part of the locus defines what we can label the 'consenting society'. Points in the region inside the curve represent consent to coercion but are not Pareto-efficient. Coercion-constrained welfare reaches a maximum at the point where $K = K_{OT}$, that is, the solutions for constrained and unconstrained welfare are equal. K_{OT} represents the level of coercion implicit in an Optimal Tax solution, when the government can impose as much coercion as it likes to set the tax rate and public provision at the levels implied by the standard optimal tax approach. Finally, coercion-constrained welfare is decreasing for values $K > K_{OT}$. For levels of coercion higher than K_{OT} the income tax rate falls below its OT size, as seen from (24.2), dragging down the level of social welfare. The downward sloping part of the trade off is thus labelled the 'masochistic society'.

We expect society to locate on the upward segment of the curve. In the present framework, higher levels of social welfare S do not unambiguously imply that society is better off in a broader context in which coercion is of vital concern. One can distinguish between narrow welfare S, as traditionally defined in optimal taxation, and broadly defined welfare, or BW, which incorporates the role of coercion in democratic society. Though we do not do so here, one could optimize a broadly defined welfare function, encompassing both the sum of utilities S and the degree of coercion required or implied by any S, $BW = B(S, K)$, in order to choose a point on the trade-off.

Although we expect society to locate on the upward sloping segment, the model does not determine a particular location without a broadly defined welfare function or another solution. The public choice literature contains possible suggestions about how to approach the choice of K. Although not worked out in a quantifiable manner, the analysis of Buchanan and Tullock (1962, chap. 6) of the optimal decision process may serve a guide to the development of a theory for determining K.[6]

Even though an optimal K is not determined here, our analysis suggests some interesting policy experiments. For a given K, we can ask what tax policies provide maximum social welfare, and how the value of K affects the choice among policies. In the next section, we reconsider the Ramsay rule for commodity taxation from such a perspective. We shall determine the nature of commodity taxation that minimizes the aggregate excess burden for any given pattern of coercion and compare the results to the traditional analysis of this problem.

The analysis also suggests the way towards formally integrating differing views about coercion in society into the design of tax blueprints.[7] For example, one might ask about longstanding problems in public finance such as the choice between income and consumption taxation when coercion is formally acknowledged and the trade-off can be explicitly determined.

7. Optimally Coercive Commodity Taxation

A longstanding problem concerns the relationship between elasticities of demand and the structure of commodity taxation required to minimize the aggregate excess burden of taxation. We re-examine this problem in our framework.

To keep the analysis as simple as possible, we assume two individuals or homogeneous groups of taxpayer-consumers $j=1, 2$ consuming two commodities $i = 1, 2$, whose prices are P_1 and P_2. Individual consumption is denoted by X_{ji}. Each taxpayer pays ad-valorem commodity taxes t_1 and t_2 on each of two consumption goods. The tax revenue collected is returned lump sum to each individual in equal amounts, denoted by $R/2$. Taxpayer 1 is assumed to pay too much tax in comparison to what he receives and vice versa for taxpayer 2.

When a coercion constraint is introduced, the taxes must be set so that the overpayment made by taxpayer 1 does not exceed a given sum K_1 and the underpayment made by taxpayer 2 must not fall below a certain level K_2. Formally,

[6] An alternative approach to determining K may rely on ideas from the contractarian literature (Reference).
[7] Buchanan and Congleton (1998) are concerned with this problem but do not do this as part of a formal welfare analysis.

$$t_1P_1X_{11}+t_2P_2X_{12}-(R/2)\le K_1$$

and

$$t_1P_1X_{21}+t_2P_2X_{22}-(R/2)\le -K_2\,.$$

Denoting the price elasticity of demand for *i* by e_i and assuming that it is identical across the two taxpayers (that is, $e_{ji}=e_i$ for j=1, 2), the total excess burden of the tax is written as

$$B=(1/2)e_1t_1^{\,2}P_1(X_{11}+X_{21})+(1/2)e_2t_2^{\,2}P_2(X_{21}+X_{22})\,.$$

The government is assumed to choose t_1 and t_2 to minimize the excess burden of commodity taxation after it secures a revenue *R* and subject to the coercion constraints described above. The budget constraint of the government is

$$t_1P_1(X_{11}+X_{21})+t_2P_2(X_{12}+X_{22})=R\,.$$

The Lagrangean of the problem is written as (where κ_1 and κ_2 denote the relevant multipliers of the coercion constraints)

$$\begin{aligned}\Lambda=(1/2)&e_1t_1^{\,2}P_1(X_{11}+X_{21})+(1/2)e_2t_2^{\,2}P_2(X_{21}+X_{22})\\&+\mu[R-t_1P_1(X_{11}+X_{21})-t_2P_2(X_{12}+X_{22})]\\&+\kappa_1\{K_1-[t_1P_1X_{11}+t_2P_2X_{12}-(R/2)]\}\\&+\kappa_2\{-K_2-[t_1P_1X_{21}+t_2P_2X_{22}-(R/2)]\}\,.\end{aligned}$$

Differentiating with respect to t_1 and t_2, denoting taxpayer's j share of consumption of commodity *i* by $w_{ji}=X_{ji}/\Sigma_i Xj_i$, and dividing the resulting equations we obtain the coercion-adjusted formula for efficient commodity taxation

$$\frac{t_1}{t_2}=\frac{\mu+\kappa_1w_{11}+\kappa_2w_{21}}{\mu+\kappa_1w_{12}+\kappa_2w_{22}}\frac{e_2}{e_1}\,. \tag{29}$$

The standard inverse elasticity formula is obtained as a special case of (29) when $\kappa_1=\kappa_2=0$, that is,

$$\frac{t_1}{t_2}=\frac{e_2}{e_1}\,,$$

which implies that in the present partial equilibrium framework the more inelastic good must be taxed more heavily. For example, if $e_2>e_1$, for efficiency it must be $t_1>t_2$. However, it is now clear from (29) that this standard result is no longer valid when coercion is taken into account. The efficient tax commodity tax rate now depends not only on the inverse of the demand elasticity but also on the level of coercion tolerated by the taxpayers; this dependence may in turn reverse the standard conclusion and require that the more elastic good must be taxed more heavily.

For example, assuming again that if $e_2 > e_1$, but that the ratio

$$p = (\mu + \kappa_1 w_{11} + \kappa_2 w_{21}) / (\mu + \kappa_1 w_{12} + \kappa_2 w_{22})$$

is lower than the elasticity ratio e_2 / e_1, then for efficiency $t_1 < t_2$, which says that the more elastic good 2 must be taxed heavier than the less elastic good 1. Although the actual size of the *p*- ratio is an empirical issue, one may expect that the higher the w_{12} and w_{22} budget shares (the more both people consume good 2), the more likely that the p-ratio is lower than the e_2/e_1 ratio and therefore that commodity 2 should be taxed more heavily than good 1, contrary to the Ramsay formula.

8. Conclusion

To be discussed by the seminar. Some notes for consideration:

- Becker (1983) has proposed a positive theory of policy outcomes in a democratic state that combines the Hicks-Kaldor potential compensation criteria with an understanding of inequalities in political influence. If the gainers from a policy action could gain more than the losers, he argues that they will be more influential in the political process (because they will spend more to influence it), unless there is some inequality in political influence in favour of the losers. Outcomes will therefore be Pareto- efficient unless inequalities in political influence intervene. We have proposed a normative theory of policy that combines the Hicks-Kaldor criterion, as well as more general social welfare objectives, with normatively desirable constraints on the extent to which losers *or* gainers *should* be coerced by the public sector.

- We have at the outset been careful to acknowledge that coercion constraints are not simply a variation of distributional objectives in social planning. To see that coercion is different from distribution, it may be useful to refer to the Bill of Rights in the U.S. constitution and other similar documents. These rights apply equally to poor and rich; they were not created with reference to income levels, but with reference to individual lives. There may of course be an interaction of criteria in the way courts apply rights in practice. And the poor may lack the resources to enforce their rights. But this is a different thing from an approach that lumps coercion together with distribution as a criterion.

- There is a great deal of talk about coercion in the literature by economists who are conservative, but almost no attempt to define coercion or to draw out its implications for fiscal structure[8]. Our approach integrates coercion as a criterion into the analysis in a way that is potentially measurable. We have suggested the nature of factors or variables that would have to be measured to decide how tax systems would be

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[8] Possibly the book by Wiseman is an exception.

structured to take coercion into account along with economic efficiency. It is an interesting question how coercion itself can be measured along with these quantities.[9]

- We can extend analysis in two directions: (a) towards social planning, by incorporating incentive compatibility constraints[10]; (b) towards social choice by adding in the determination of coercion constraints as part of the choice of an optimal decision process.

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[9] If we argue that coercion applies primarily to individuals, in measuring hte degree of coercion we will have to use an approach that somehow measures "losses of economic freedom" for different individuals.

[10] ***On (a) - incentive compatibility(IC).***

IC constraints cannot be defined independently of the degree of cercion, since these are defined only by assuming something about what the state can know about a person, or do with a given knowledge. Thus the degree of coercion allowed determines what the IC constraints will be, though such matters are not acknowledged in the OT literature. One might give everyone an enforced IQ test and tax on that basis.

Once the IC constraints are determined however, there are several cases to consider. Assume there are two groups, rich and poor:

case 1: Coercion constraints (CC) bind on rich and poor first. IC constraints are then irrelevant. This case is that considered above.

case 2: IC binds on both first. This is the standard OT problem with IC constraints. No need to do this again.

case 3: IC binds first on the rich, and CC on the poor. (This parallels the case usually considered in the OT literature where IC binds on the rich, not on the poor, and the problem is to redistribute to the poor from the rich.) Here there are two sub cases: (i) the poor want more public services than are provided in the social planning solution, and (ii) they want less.

If (i), then taxes will have to be raised on the poor in the OC solution, and the IC constraint on the rich prevents more taxes from being levied on the rich. This means that mimicking a poor person is less desirable than before (compared to the OT solution), and the IC constraint is relaxed relative to the OT problem with IC. The rich can and will be taxed more in the coercion-constrianed social planning solution (the OC solution). In short, in OC, the IC constraint and the CC interact.

If (ii), the poor want less public spending than in the OT solution. The poor will be taxed less in the OC problem than in the OT problem with IC constraints, making mimicking by the rich more interesting to them.

case 4: IC binds on the poor, and a CC constraint on the rich. We don't want to tax the rich anymore so as to redistribute. Redistribution will be less than in the OT solution, where teh IC constraint is already binding on the rich.

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Figure 1
Coercion Measured by the Level of a Public Good, When $G^* < G^e$

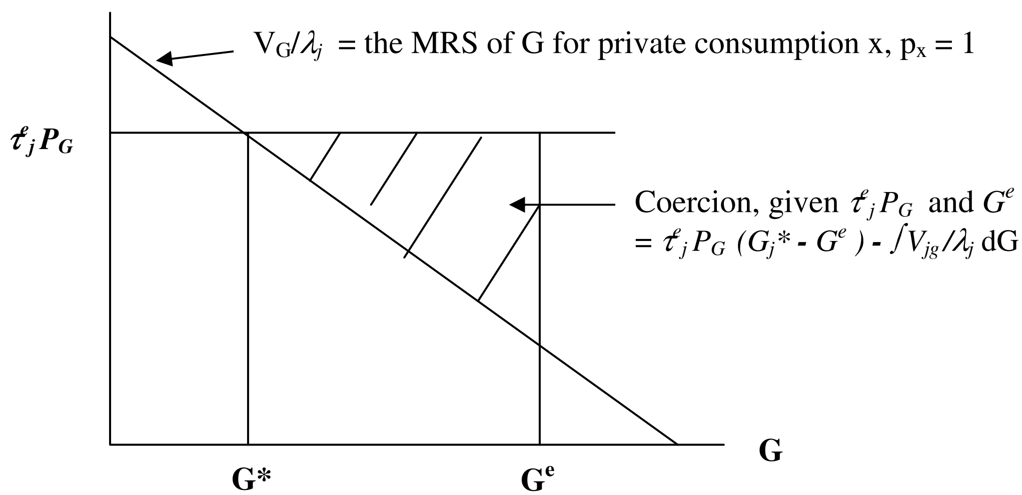
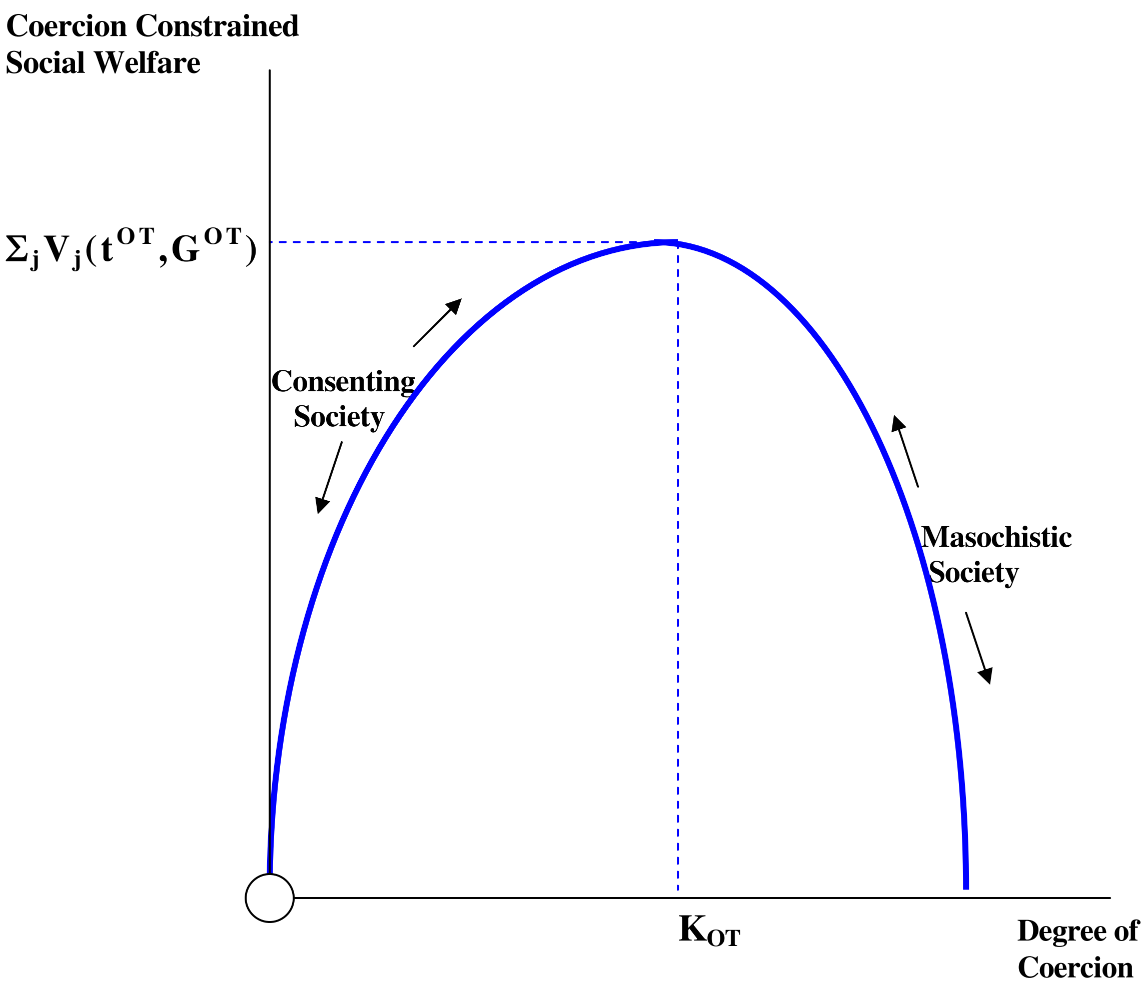


Figure 2
The Coercion - Welfare Frontier



$\Sigma_j V_j(t^{OT}, G^{OT})$: Social welfare (sum of individual utilities) with Optimal Tax solution.

At the origin $K=0$, the tax rate takes the value of zero and the planner's social welfare maximization problem is not defined.

Appendix: A brief note on the importance of coercion in the history of public finance (tentative)

Here we briefly survey previous normative approaches from both social planning and public choice perspectives to public finance to expose the role of coercion - or its absence - in the history of the field. (We note that *The New Palgrave* contains no entry under 'coercion'.)

(i) *Social planning (and Optimal Taxation)* - see Mirrlees (1970) and Atkinson and Stiglitz 1980) - maximizes room for the normative policy analyst because all political institutions are ignored. The fact that collective choice is a pre-requisite for the collective action that is required to generate public services (which cannot be efficiently provided by private markets) is the central problem with this framework. Policy choices emerging from this approach - which optimize what we call 'narrow social welfare' - may not be consistent with feasible or acceptable collective choice mechanisms. There is no way to tell from the content of OT analysis itself. This problem often leads to the incorporation of politics in an informal and ad hoc manner by astute economic policy advisors, a process which, as Sandmo (1984, 116) points out, increasingly robs welfare analysis of its normative content[11]. As Sandmo puts it: *If the economist were to accept any kind of "political constraint" on the tax system as true constraints on economic policy, much of the prescriptive power of welfare analysis would clearly be lost*.

(ii) *Wicksell / Lindahl unanimity* is a normatively attractive standard that encompasses both an efficient allocation and a desirable collective choice mechanism, where efficient outcomes necessarily involve the complete absence of coercion.[12] Unanimity, or even Wicksell's (1896) approximate unanimity, ignores the cost of decision making and thus is not descriptive of the real world. Almost every policy outcome in a democracy is undesirable by this standard because it will involve some coercion. (Inefficiency will also occur because of preference revelation and free-rider problems, just as in standard OT approach). Lindahl (1919) went further and proposed a process for actually achieving a solution exhibiting efficiency *and* the absence of coercion. But his process requires, as he understood, an equal distribution of political power among citizens or among their representatives.

(iii) Henry Simons (1938) argued for a broad- income tax with few special provisions. His idea is that this would limit government involvement in the economy, which he regarded as necessary for both efficiency and personal freedom. Subsequent applications of this idea to taxation, such as tax blueprints that implement a broad based income tax, or a braid based consumption tax (see Hettich and Winer 1985 for further discussion) more or less ignored Simon's original motivation which was intimately connected with coercion in a democratic society.

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[11] See also the Boadway-Hettich debate in Winer and Shibata (2002).

[12] *From the point of view of general solidarity...parties and social classes should...share an expense from which they receive no great or direct benefit. Give and take is a firm foundation of lasting friendship... It is quite a different matter, however, to be forced so to contribute. Coercion is always an evil in itself and its exercise, in my opinion, can be justified only in cases of clear necessity.* Knut Wicksell (1896, 1958, 90)

(iv) Buchanan and Tullock (1962) optimize coercion by choice of the k-majority rule decision procedure, given the costs of coercion (which are decreasing in the percent k required for a majority) and the costs of decision making (which are increasing in k). The optimal k need not equal 50%, and whatever it is, implies an certain, nonzero, degree of coercion is optimal. Since they focus on the decision procedure itself, they do not inform us about the nature of policy that is social welfare optimizing, or what the trade-off between coercion and narrowly defined welfare might be.

(v) In *The Power to Tax (1980),* Brennan and Buchanan provide a normative framework in which the state is assumed to have and use a monopoly on coercion to oppress taxpayers. This can be viewed as a complement to the Scandinavian tradition, which is not concerned with the principal-agent problem for citizens vis a vis government. (J.S Mill took a similar position - reference). Policy making thus starts from the position that one needs to defend against the Leviathan-like power of the state. Brennan and Buchanan (p 88) show that an Optimal Tax plan for taxation is analytically equivalent to a program designed to maximize government size and excess burden. That is, they show that the tax structure that maximizes revenue, and the one that maximizes social welfare are the same, though of course the OT program involves a fixed level of taxation.

(vi) Usher (1982) asks about how we ought to amend market outcomes through collective choice to produce politically acceptable distributions of income and well-being. (Expand)

(vii) *The Representation theorem (e.g.,* Coughlin/Nitzan 1981, Hettich and Winer1999, chapter 6*)* in a spatial voting framework is another way of linking public choices to normatively attractive policy outcomes in a democratic society. The Representation theorem has two parts: (i) policy outcomes are general equilibrium outcomes within a broad economic-political system operating under some form of majority rule; (ii) policy outcomes, under certain circumstances, are Pareto efficient. From the perspective of a concern with coercion, this approach goes to far because it implicitly assumes that democratic institutions are acceptable, along with the resulting political equilibria.

(viii) Breton (1996) attempts to define coercion along the lines here, though we think not as clearly. The main contribution of the book is to link the operation of political institutions to what he calls the 'Wicksellian connection' (or lack thereof) between what people pay and what they get. For example, the institutions of parliamentary democracy are contrasted with those of Congressional government: in the former, there is a Minister of Finance, backed by the Prime Minister and aided by budgetary traditions like budget secrecy and cabinet solidarity that enable the Minister to keep spending and taxing inline with each other to a grater extent than is possible in a Congressional system of continual checks and balances.

(ix) *The constitutional approach*. Buchanan and Congleton (1998) construct a normative framework consistent with Wicksell (1896) and Simons (1938). They rule out all coercion regardless of inefficiency. Their tax rule is essentially flat taxation on income with no demo-grant, so no one is tempted to using the fiscal system to coercively redistribute. In our view this goes too far - some coercion may be desirable. But here it is a matter of judgment how far to go, and their analysis could be viewed as a limiting case of the present one.

(x) Boadway and Sato (2002, in Winer and Shibata 2002)) provide a thoughtful analysis which informs the present paper to some extent. They want to include existing or actual constraints into social planning analysis in a limited, but structured manner. A problem in their framework remains that of establishing an ideal or standard of reference. Which constraints do we accept and which not? What form of idealism do we wish to establish for social planning? The Boadway/Sato analysis doesnt address this issue.